

PhD Seminars 2004/2005: Prof. R. Brown Semester 3

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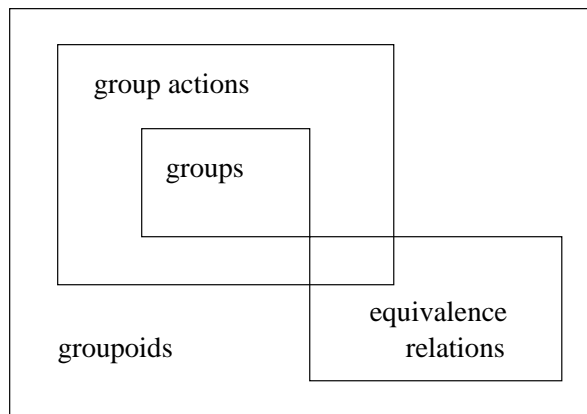
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1 Seminar 1: 19th April 2005

1.1 Groupoid Atlases: Global Actions, Kripke Frames

Ronnie's Note: "I can give a talk (prepared in the usual style) on 'Groupoid Atlases' as an introduction, which would be helpful for me to prepare an abstract (by April) and paper (by June) for the Ehresmann conference in October in Amiens".



- Groupoids add a spacial component to groups (i.e. \exists objects).
- We then talk about "groupoids which patch".
- Bak's global actions considered a family of group actions (G_α, X_α) .
- A groupoid atlas is a special case of a global action.
- Authors: A.B., R.B., G.M., T.P.

Definition 1.1 (Groupoid Atlas)

- We have an indexing set Φ with a reflexive action \leq on it.
- Each $\alpha \in \Phi$ is a groupoid G_α with object set X_α .
- For all $\alpha \leq \beta$ we have a groupoid morphism $\varphi_{\alpha\beta} : G_\alpha | X_\alpha \cap X_\beta \longrightarrow G_\beta | X_\alpha \cap X_\beta$ ($\varphi_{\alpha\alpha} = 1$).

Axiom: If $\alpha \leq \beta$ and $g \in G_\alpha$ has $sg \in X_\alpha \cap X_\beta$, then $tg \in X_\alpha \cap X_\beta$ (i.e. $X_\alpha \cap X_\beta$ is a "union of components" of $G_\alpha | X_\alpha \cap X_\beta$).

2 Seminar 2: 21st June 2005

2.1 Leading towards homotopies and tensor products for crossed complexes

Ronnie's Note: "The point is that to define $C \otimes D$ for crossed complexes C, D you have to understand the tensor product $(M \otimes N, G \times H)$ of groupoid modules $(M, G), (N, H)$, so this category and its monoidal closed structure is well worth discussing first".

Two aspects make crossed-complexes useful:

- (1) Generalised van Kampen Theorem: local-to-global, non-abelian;
- (2) Homotopies and tensor products.

Very short recap of the seminar:

- (1) Background in tensor products.
 - Cartesian Closed Categories.
 - Abelian Groups.
- (2) Crossed Complexes.
- (3) *Comment:* Exponential laws are a good thing!!

Next time, we will discuss

$$\text{Crs}(C \otimes D, E) \cong \text{Crs}(C, \text{CRS}(D, E)).$$

3 Seminar 3: 23rd June 2005

3.1 Leading towards homotopies and tensor products for crossed complexes, part 2

Very short recap of the seminar:

(1) Chain Complexes (of abelian groups).

- Chn = category of chain complexes.
- Motivation: $\text{Chn} \simeq \text{SimpAbGp}$ (Dold-Kan).
- There is a conflict between information conceptually understandable and minimal in content.

(2) Crossed Complexes.

- Why consider Crs , the category of crossed complexes? The answer is the functor

$$\text{FTop} \xrightarrow{\Pi} \text{Crs},$$

where FTop = Filtered Topological spaces and Π is defined homotopically.

- RB's work with Philip Higgins 1974-2004:
 - Π satisfies a GvKT so that in some cases it can be directly computed.
 - FTop is monoidal closed.
 - Crs is monoidal closed and there exists a natural transformation

$$\eta : \Pi X_* \otimes \Pi Y_* \longrightarrow \Pi(X_* \otimes Y_*)$$

which in some good cases is an isomorphism.

– $R(X_*)_n = \text{FTop}(I_*^n, X_*)$.

- Problem: Although we have $\text{Crs}(C \otimes D, E) \cong \text{Crs}(C, \text{CRS}(D, E))$ and the description of $\text{CRS}(D, E)$ is explicit, $C \otimes D$ is given by generators and relations and so is 'cloudy'.
- Remark: We use cubes for conjectures and proof, and crossed complexes for calculations!