PhD Seminars 2004/2005: Dr. C. D. Wensley Semester 3

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Contents

1 Seminar 1: 3rd May 2005 **2**

1.1 Automata for Deciding Normal Form 2

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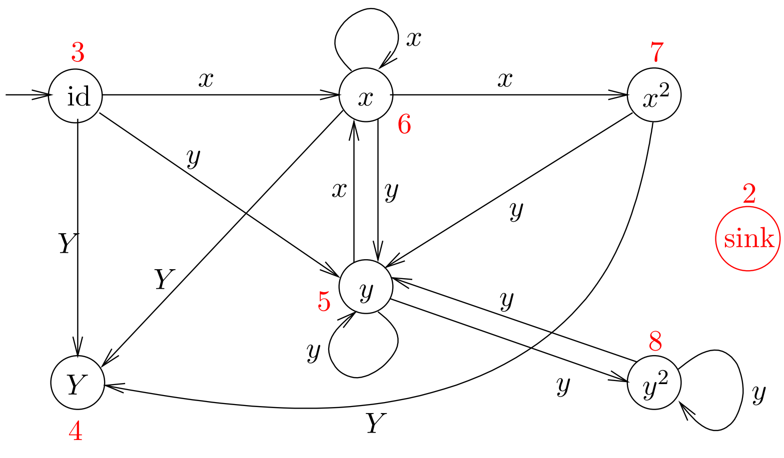
1.1 Automata for Deciding Normal Form

Consider the group $T = \langle x, y \mid x^3 = y^2 \rangle$, with $X > x > Y > y$. We will use the wreath product ordering, defined as follows: Let w_1 an w_2 be two words with no common prefix. Let a_i be the maximal letter in w_i $(i = 1, 2)$. Then $w_1 > w_2$ if $a_1 > a_2$; else let n_i be the number of occurrences of a_i in w_i $(i = 1, 2)$. Then $w_1 > w_2$ if $n_1 > n_2$; else write $w_1 = u_0 a_1 u_1 a_1 u_2 a_1 \dots a_1 u_{n_1}$ and $w_2 = v_0 a_1 v_1 a_2 v_2 a_1 \dots a_1 v_{n_1}$, and return $(u_0 > v_0)$? Here is the wreath product complete rewrite system for T:

$$\{Yy \to 1,\ yY \to 1,\ X \to x^2Y^2,\ x^3 \to y^2,\ y^2x \to xy^2,\ Yx \to yxY^2\}.$$

Consider that we want an automaton which takes (padded) words in the monoid and returns true if the word is in normal form and false if the word is reducible (so that the word contains a left hand side of a rule as a subword). Here is such an automaton: States = all proper prefixes of the left hand sides of the rules, a state for the identity word, an accept state, and a sink state; the only accept state is the sink state; the only initial state is the state corresponding to the identity word; we have a transition from a state labelled by a word u to the sink state via a letter a if the word ua is a left hand side in the complete rewrite system, we have a transition from a state labelled by a word u to a state labelled by a word s via a letter a if s is a suffix of ua. Here is the automaton for the above complete rewrite system (minus the transitions to and from the sink state for clarity) together with its transition table.



	1	2	3	4	5	6	7	8
	accept	sink	id	Y	y	x	x^2	y^2
y		[2]	[5]	[2]	[5,8]	[5]	[5]	[5,8]
Y		[2]	[4]	[4]	[2]	[4]	[4]	[2]
x		[2]	[6]	[2]	[6]	[6,7]	[2]	[2]
X		[2]	[2]	[2]	[2]	[2]	[2]	[2]

By applying the subset construction to the above automaton, we obtain the following diagram from which we can obtain a deterministic finite state automaton. The complement of this automaton then gives us the desired automaton which accepts all irreducible words.