PhD Seminars 2003/2004: Other Seminars in Semester 3

Gareth Evans

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1 Seminar 1: Philippe Malbos (16th April 2004)

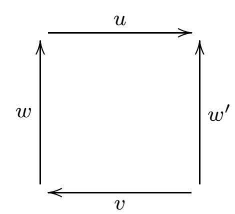
1.1 Finiteness Conditions of Squires

Consider a monomial M. M is of finite derivation type (FDT) iff the natural system $\pi(S)$ is finitely generated ($S = \langle X \mid R \rangle$).

Proposition 1.1 *There is an isomorphism between $\pi(S)$ and $H(S)$, the homotopical syzygies.*

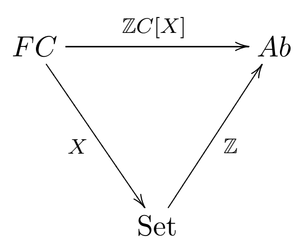
The abelian resolution of Squires is a small category C. Further, FC, the category of factorisations, has objects = $\mathrm{Mor}(C)$ and morphisms

$FC(w, w') \ni$



in C.

An abelian natural system is a functor $D: FC \to Ab$. If $\lambda: FC \to \mathrm{Set}$ is a natural system, the free abelian natural system $\mathbb{Z}C[X]$ is as follows.



If $\langle X \mid R \rangle$ is a (normalised) complete presentation of C, we have a substitution in $Nat(C)$. Let w be an arrow in C, and recall that $\mathbb{Z}C: FC \to Ab$, $w \mapsto \mathbb{Z}[(u_1, u_2) \mid u_1 u_2 = w]$. We say that R is normalised if for each $A = (\ell, r) \in R$ we have (i) r is irreducible; (ii) ℓ is only reducible by A. In this situation there are no 'middle type' critical pairs.

Proposition 1.2 (Squires) *Every complete rewrite system has an equivalent normalised rewrite system.*

We can construct the free crossed module generated by $\langle X \mid R \rangle$ by recalling that a crossed module in Cat is a category enriched in groupoid.

Definition 1.3 A rewriting system $\langle X \mid R \rangle$ is said to be of finite derivation type (FDT) if the free crossed module $C(R) \Longrightarrow X^*$ has a finite homotopy base, i.e. $P^2(\mathcal{C}) = \{\text{parallel 2-cells in } \mathcal{C}\}$. It follows that there is a finite $B \subseteq P^2(\mathcal{C})$ such that $P^2(\mathcal{C}) = \cong_B$.

Proposition 1.4 *If $\langle X \mid R\rangle$ is normalised and complete then $\mathcal{C}$ has homotopy base*

$$B = \left\{ \begin{array}{ccc} & \bullet & \\ {}^{e_1}\swarrow & & \searrow^{e_2} \\ \bullet & & \bullet \\ {}_{e_2\backslash e_1}\searrow & & \swarrow_{e_1\backslash e_2} \\ & \bullet & \end{array} \;\middle|\; \langle e_1, e_2\rangle \in P \right\},$$

where P is the set of critical pairs.

Remark 1.5 The free crossed module $\mathcal{C}$ is abelian — for each $p, q \in \mathcal{C}_0$, $\mathcal{C}(p, q)$ is an abelian groupoid. It follows that $\exists I : FC \to Ab$ such that there is an isomorphism in $Nat(X^*)$ given by $I_H \cong \mathrm{Aut}^{\mathcal{C}}$ ($\hat{w} \mapsto \mathrm{Aut}^{\mathcal{C}}_{\hat{w}}$ is the set of automorphisms on $\hat{w}$).