Thesis Summary

The theory of Gröbner Bases originated in the work of Buchberger and is now considered to be one of the most important and useful areas of symbolic computation. A great deal of effort has been put into improving Buchberger's algorithm for computing a Gröbner Basis, and indeed in finding alternative methods of computing Gröbner Bases. Two of these methods include the Gröbner Walk method and the computation of Involutive Bases.

By the mid 1980's, Buchberger's work had been generalised for noncommutative polynomial rings by Bergman and Mora. This thesis provides the corresponding generalisation for Involutive Bases and (to a lesser extent) the Gröbner Walk, with the main results being as follows.

(1) Algorithms for several new noncommutative involutive divisions are given, including strong; weak; global and local divisions.
(2) An algorithm for computing a noncommutative Involutive Basis is given. When used with one of the aforementioned involutive divisions, it is shown that this algorithm returns a noncommutative Gröbner Basis on termination.
(3) An algorithm for a noncommutative Gröbner Walk is given, in the case of conversion between two harmonious monomial orderings. It is shown that this algorithm generalises to give an algorithm for performing a noncommutative Involutive Walk, again in the case of conversion between two harmonious monomial orderings.
(4) Two new properties of commutative involutive divisions are introduced (stability and extendibility), respectively ensuring the termination of the Involutive Basis algorithm and the applicability (under certain conditions) of homogeneous methods of computing Involutive Bases.

Source code for an initial implementation of an algorithm to compute noncommutative Involutive Bases is provided in Appendix B. This source code, written using ANSI C and a series of libraries (AlgLib) provided by MSSRC, forms part of a larger collection of programs providing examples for the thesis, including implementations of the commutative and noncommutative Gröbner Basis algorithms; the commutative Involutive Basis algorithm for the Pommaret and Janet involutive divisions; and the Knuth-Bendix critical pairs completion algorithm for monoid rewrite systems.