Thesis Corrections

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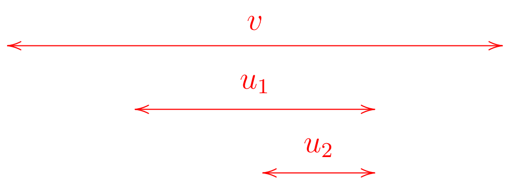
- Page 3: The second paragraph should be as follows (thanks to an anonymous referee):
    - As Gröbner Bases became popular, researchers noticed a connection between Buchberger's ideas and ideas originating from the Janet-Riquier theory of Partial Differential Equations developed in the early 20th century (see for example [44]). This link was completed for commutative polynomial rings by Gerdt and Blinkov [25] when they gave an algorithm to compute an *Involutive Basis* that provides an alternative way of computing a Gröbner Basis.
- Page 5: The third paragraph should be as follows (thanks to an anonymous referee):
    - The final piece of the jigsaw is to mirror the application of Gerdt and Blinkov's Involutive methods to the noncommutative case.
- Page 10: The definition of a rng R should contain the following additional axiom:
    - $r_1 \times r_2 \in R$ for all $r_1, r_2 \in R$ (closure).
- Page 12: An ideal should be defined in terms of a subrng, not a subring, so that the unit polynomial need not automatically be a member of any polynomial ideal.
    - If S is a subset of a rng R that is itself a rng under the same binary operations of addition and multiplication, then S is a *subrng* of R.
    - Let $\mathcal{R}$ be an arbitrary commutative ring. An *ideal* J in $\mathcal{R}$ is a subrng of $\mathcal{R}$ satisfying the following additional condition: $jr \in J$ for all $j \in J$, $r \in \mathcal{R}$.
    - Let $\mathcal{R}$ be an arbitrary noncommutative ring.

    - A *left (right) ideal* J in $\mathcal{R}$ is a subrng of $\mathcal{R}$ satisfying the following additional condition: $rj \in J$ ($jr \in J$) for all $j \in J$, $r \in \mathcal{R}$.
    - A *two-sided ideal* J in $\mathcal{R}$ is a subrng of $\mathcal{R}$ satisfying the following additional condition: $r_1 j r_2 \in J$ for all $j \in J$, $r_1, r_2 \in \mathcal{R}$.

- Page 105: The third paragraph should be as follows:
  - More formally, let u_1 and u_2 be two monomials over a noncommutative polynomial ring, and assume that u_2 is a conventional divisor of u_1, so that $u_1 = u_3 u_2 u_4$ for some monomials u_3 and u_4....

- Page 112: The final paragraph should be as follows:
  - In the noncommutative case, we cannot hope to produce a carbon copy of the above results because a finitely generated ideal may have an infinite Gröbner Basis, ...

- Page 138: The first part of the proof of Proposition 5.5.22 should be as follows:
  - To prove that the strong left overlap division is a strong involutive division, we need to show that the three conditions of Definition 5.1.6 hold.
    - **Disjoint Cones Condition**
      Let $\mathcal{C}_{\mathcal{S}}(u_1, U)$ and $\mathcal{C}_{\mathcal{S}}(u_2, U)$ be the involutive cones associated to the monomials u_1 and u_2 over some noncommutative polynomial ring $\mathcal{R}$, where $\{u_1, u_2\} \subset U \subset \mathcal{R}$. If $\mathcal{C}_{\mathcal{S}}(u_1, U) \cap \mathcal{C}_{\mathcal{S}}(u_2, U) \neq \emptyset$, then there must be some monomial $v \in \mathcal{R}$ such that v contains both monomials u_1 and u_2 as subwords, and (as placed in v) both u_1 and u_2 must be involutive divisors of v. By definition of $\mathcal{S}$, either (as placed in v) u_1 must be a suffix of u_2, or u_2 must be a suffix of u_1. Thus, assuming (without loss of generality) that $\deg(u_1) > \deg(u_2)$, we are able to draw the following diagram summarising the situation.



      For $\mathcal{S}$ to be strong, we must have $\mathcal{C}_{\mathcal{S}}(u_1, U) \subset \mathcal{C}_{\mathcal{S}}(u_2, U)$ (it is clear that $\mathcal{C}_{\mathcal{S}}(u_2, U) \not\subset \mathcal{C}_{\mathcal{S}}(u_1, U)$ because $u_2 \notin \mathcal{C}_{\mathcal{S}}(u_1, U)$). This can be verified by proving that a variable is right nonmultiplicative for u_1 if and only if it is right nonmultiplicative for u_2...

Spotted any further errors? Send them along to `postgareth@hotmail.com`.